Real-time identification of nonlinear multiple-input–multiple-output systems with unknown input time delay using Wiener model with Neuro-Laguerre structure

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Summary

In this article, a real-time block-oriented identification method for nonlinear multiple-input–multiple-output systems with input time delay is proposed. The proposed method uses the Wiener structure, which consists of a linear dynamic block (LDB) followed by a nonlinear static block (NSB). The LDB is described by the Laguerre filter lattice, whereas the NSB is characterized using the neural networks. Due to the online adaptation of the parameters, the proposed method can cope with the changes in the system parameters. Moreover, the convergence and bounded modeling error are shown using the Lyapunov direct method. Four practical case studies show the effectiveness of the proposed algorithm in the open-loop and closed-loop identification scenarios. The proposed method is compared with the recently published methods in the literature in terms of the modeling accuracy, parameter initialization, and required information from the system.

KEYWORDS

input time delay, Laguerre-neural structure, Lyapunov-based adaptation, real-time identification, Wiener model

1 | INTRODUCTION

System identification plays an important role especially in the cases where modeling based on mathematical, physical, chemical, and the other laws is almost impossible to obtain or has a very complicated procedure. For this reason, input-output signals that are obtained from the system are used to determine the parameters of the model. The block-oriented identification methods have been proposed in the literature to model nonlinear systems. These methods are more convenient as compared to other identification methods. This is mainly due to the fact that they are composed of a combination of linear dynamic block (LDB) and nonlinear static block (NSB). The Wiener and Hammerstein models are the simplest among the block-oriented methods. In the Wiener model, an LDB is followed by an NSB, whereas the blocks in the Hammerstein model are in the reverse order. These structures can describe a large number of

Abbreviations: BIBO, bounded-input bounded-output; CSTR, continuous-stirred tank reactor; LDB, linear dynamic block; LTI, linear time-invariant; MIMO, multiple-input multiple-output; MISO, multiple-input single-output; MLP, multilayer perceptron; NN, neural network; NSB, nonlinear static block; PE, persistency of excitation; RLS, recursive least squares; RMSE, root-mean-squared error.
nonlinear plants with desired accuracies. Among them are chemical plants, soft sensor systems, thermodynamic engines, physiological systems, fuel cells, power amplifier, and tubular reactor, just to name a few.

Input time delay often arises in the mass or energy transportation systems and in the flow-temperature composition phenomena in the process control systems. The measurement delay, which appears as the input time delay, is also common in the industrial plants. Some of the open problems that occur with the presence of the delay phenomenon have been discussed in the work of Richard. The problem of modeling of a subcutaneous tissue, which involves a known input time delay, is investigated in the work of Tärnik et al. In the work of Najafi et al, using the wavelet transformation, a method for estimation of the input time delay has been proposed for linear systems with unknown input time delay. Rad et al introduce a correlation approach for approximation of the input time delay. A modified recursive least-squares algorithm for the online identification of linear systems with input time delay is proposed in the work of Ren et al. In recent years, simultaneous identification of the time delay and the dynamic parameters of the systems have been studied by researchers. In this regard, Chen et al have proposed a combined method from instrumental variable and gradient search to estimate the plant time delay.

However, the problem of estimation of the input time delay in nonlinear systems into the block-oriented structures has been less pursued by researchers. An identification method based on Wiener models has been devised for a restricted class of nonlinear processes with input time delay in the work of Huang and Lee. One of the newest and most significant works that employs the Hammerstein model has been proposed in the work of Heuberger et al for identification of nonlinear systems with input time delay.

A basic property of the Laguerre orthonormal functions is that they are uniformly bounded in the frequency domain. This leads to the bounded identification error for the linear systems. It is shown that, using the proper selection of the Laguerre filter parameter, smaller modeling error can be obtained as compared to the ARX modeling. This parameter can be obtained using a priori knowledge of the system time constant. Furthermore, the identification of linear systems knowing only an approximation of the input time delay is a distinguished aspect of the Laguerre filters. In the work of Bouzrara et al, a new representation of the ARX models by filtering the input-output signals using the Laguerre functions has been proposed. In the works of Mäkilä and Wahlberg and Mäkilä, an approximation of the stable infinite dimensional linear systems using a class of orthonormal exponential functions has been studied. Time-delay systems are used to demonstrate the modeling capability of the Laguerre and Kautz filters. The Kautz filters have complex poles, whereas the poles of the Laguerre filters are real. Researchers have employed the aforementioned capability of the Laguerre filters to identify the nonlinear systems using Wiener models. The kernel-based algorithms have been used to reach this goal; e.g., polynomial functions, support vector machine regression, wavelet neural network (NN), and extreme-learning machine are some of the most important references.

In this paper, nonlinear multiple-input–multiple-output (MIMO) systems with time delay in the input lines are identified using the Wiener model with good accuracies. The proposed method uses the multilayer perceptron (MLP) NN as the NSB, whereas the LDB is composed of the Laguerre filters. In other words, the advantages of the Laguerre filters (to represent the linear and input-delayed behavior of the system) and the MLP NN (to approximate the nonlinearity of the system) are employed simultaneously in one model. The Laguerre filter parameter is determined using the step response of the system in the offline mode. Then, knowing the Laguerre filter parameter, the state and the input matrices are determined. On the other hand, in the online mode, the output vector of the LDB and the weights of the MLP NN are calculated using the system's input-output data via the recursive least squares (RLS) and the Lyapunov-based procedures, respectively.

In order to show effectiveness of the proposed technique, four practical case studies are utilized. The first example is used to demonstrate the Laguerre filter characteristic to approximate linear systems with input time delay. This ability is shown analytically in Section 4. The next two examples are introduced to demonstrate capability of the Laguerre filters and the MLP NN to model nonlinear systems with input time delay. The analytical study of this property is shown in Section 4 using the Lyapunov direct method. The last example shows the applicability of the proposed algorithm in the identification of nonlinear systems in closed-loop control scenarios. The simulation examples demonstrate the superiority of the proposed block-oriented identification method as compared to the recently proposed algorithms in the literature in terms of modeling accuracy, parameter initialization, changes in the system parameters, and required information from the system.

The main characteristics and contributions of the proposed method can be summarized as follows:

- Nonlinear MIMO systems with the input time delay are identified in the block-oriented Wiener structure.
- The advantages of the Laguerre filters to estimate the delayed behavior of the system and the MLP NN to approximate the nonlinearity of the system are employed in the Wiener model.
• The proposed method can cope with changes in the system parameters due to the online adaptation of the parameters (the third case study in the Simulation section).
• The Laguerre filter characteristic to approximate linear systems with input time delay is shown analytically (Theorem 1).
• Using the Lyapunov direct method, the analytical study about the convergence of the proposed Wiener model for modeling of nonlinear systems with input time delay is demonstrated (Theorem 2).
• The superiority of the proposed method in terms of the modeling accuracy, parameter initialization, and required information from the system is shown in the practical simulation examples and compared to the recently proposed algorithms in the related literature.
• It is shown that the proposed method can also be employed for the closed-loop control strategies, where identification of the system is required.

This paper is organized as follows. Section 2 briefly explains the discrete-time Laguerre filter representation and the MLP NN. Section 3 describes the identification problem and the proposed modeling algorithm. Convergence analysis is presented in Section 4. Simulation examples are given in Section 5. Section 6 concludes this paper.

2 | PRELIMINARIES

2.1 | Discrete-time Laguerre filter

A strictly proper discrete-time transfer function $G(z)$ that is analytic in $|z| > 1$ and continuous in $|z| ≥ 1$ can be represented as:

$$G(z) = \frac{\hat{y}(z)}{u(z)} = \sum_{i=1}^{N} c^{(i)} L^{(i)}(z),$$

where $\{c^{(i)}\}_{i=1}^{N}$ is a real sequence; $u(z)$ and $\hat{y}(z)$ are the input and output signals, respectively; $N$ is the number of the Laguerre filters; and $L^{(i)}(z)$ is the $i$th order Laguerre filter that can be represented in the $z$-domain as

$$L^{(i)}(z) = \sqrt{(1-a^2)T_s} \frac{(1-az^{i-1})}{(z-a)^i}, \quad i = 1, \ldots, N,$$

in which $T_s$ is the sampling time, and $a$ is the parameter of the Laguerre filter that belongs to the open interval $(-1,1)$.

Consider the state vector of the Laguerre filter as

$$l(k) = [l^{(1)}(k) \ l^{(2)}(k) \ \cdots \ l^{(N)}(k)]^T,$$

where $l^{(i)}(k)$ is the output of the $i$th order Laguerre filter at time instant $k$. Then, the state-space realization of the model can be described as follows:

$$\begin{cases} l(k+1) = A(a) l(k) + B(a) u(k) \\ \hat{y}(k) = C(k) l(k), \end{cases}$$

where the state-space matrices can be defined as

$$A(a) = \begin{bmatrix} a & 0 & 0 & \cdots & 0 \\ (1-a^2) & a & 0 & \cdots & 0 \\ -1(1-a^2) & (1-a^2) & a & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^{N-1} a^{N-2} (1-a^2) & (-1)^{N-1} a^{N-3} (1-a^2) & \cdots & a \end{bmatrix},$$

$$B(a) = \begin{bmatrix} \sqrt{(1-a^2)T_s} - a \sqrt{(1-a^2)T_s} & \cdots & (-a)^{N-1} \sqrt{(1-a^2)T_s} \end{bmatrix}^T,$$

$$C(k) = \begin{bmatrix} c^{(1)}(k) \ c^{(2)}(k) \ \cdots \ c^{(N)}(k) \end{bmatrix}.$$

A useful method to define the parameter of the Laguerre filter is:

$$a = e^{-\frac{T_s}{2}}.$$
where \( T \) is the time constant of the system. Hence, based on (5), this parameter is limited to \( 0 < a < 1 \). The main advantage of (5) is that \( a \) can be computed in terms of the sampling time and the time constant of the system. This will be explained further in Section 4.

In this manuscript, the LDB of the Wiener model is determined using a sequence of the Laguerre filters defined in (3) and the parameter defined in (5). By selecting the Laguerre filter parameter in the neighborhood of the actual pole of the system, faster convergence rate can be obtained.\(^{30}\) For the time delay, a proper estimation can be obtained due to the orthonormal property of the Laguerre functions similar to the Padé approximation.\(^{31}\)

### 2.2 MLP neural network

In this paper, the MLP NN is considered to describe the NSB in the Wiener structure. The employed NN is composed of an input layer, one hidden layer, and an output layer. The weights of the MLP are updated in an online procedure using the Lyapunov direct method to guarantee stability of the proposed method. Moreover, the convergence of the outputs of the proposed model is assured.

The identification algorithm will be given in the next section.

### 3 PROBLEM DEFINITION AND IDENTIFICATION PROCEDURE

The nonlinear MIMO system is considered as follows:

\[
\begin{align*}
\dot{x}(t) &= F(x(t)u(t - \tau)) \\
y(t) &= G(x(t)),
\end{align*}
\]

where \( F \) and \( G \) are nonlinear functions that are assumed to be continuous Lipschitz; \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^p \), and \( y \in \mathbb{R}^m \) are the state, input, and output vectors of the system, respectively; and \( \tau \) is the time delay. The nonlinear system represented in (6) is assumed bounded-input–bounded-output (BIBO) stable, which is necessary for the open-loop identification methods. On the other hand, the input signal should be sufficiently rich in order to excite all of the nonlinear modes of the system and to implement the RLS algorithm in the LDB part. The richness condition can be translated into the persistency of excitation (PE) condition. This means that the input signal should be PE with the order of more than the number of Laguerre filters.\(^{32}\) Moreover, the input time delay (\( \tau \)) is bounded and constant.

In this manuscript, it is proposed that system (6) can be modeled using a Wiener model that is composed of \( m \) subsystems (where \( m \) is the number of system outputs), defined as follows:

\[
\begin{align*}
I_i(k + 1) &= A_i(a_i)I_i(k) + B_i(a_i)u(k) \\
v_i(k) &= C_i(k)I_i(k) \\
\hat{y}_i(k) &= w_i^T(k)\Phi(v(k)),
\end{align*}
\]

where \( A_i(a_i), B_i(a_i), \) and \( C_i(k) \) (\( i = 1, \ldots, m \)) are defined the same as in (4). However, \( B_i(a_i) \) are composed of \( p \) columns due to the multiple inputs, \( \Phi_i \in \mathbb{R}^{p_i} \) is the vector of the activation functions in the NN, where \( H_i \) is the number of hidden neurons, and \( \Phi_i = [\Phi_{1i}(k), \ldots, \Phi_{H_i}(k)]^T \) and \( v(k) = [v_1(k), \ldots, v_m(k)]^T \) are the vectors of the synaptic weights and the inputs to the \( i \)th NN, respectively. To form \( A_i(a_i) \) and \( B_i(a_i) \), the Laguerre filter parameter \( (a_i) \) should be calculated for each subsystem using Equation (5). The output vector \( (C_i(k)) \) for each subsystem is determined using the RLSs method. Other information about the proposed method will be presented subsequently.

Figure 1 shows the block diagram of the proposed model. Simultaneous employment of the Laguerre filters to describe the linear dynamics with input time delay in an orthogonal space and MLP NN ability to approximate the nonlinearity of the system are considered in the proposed Wiener model. Since this structure has a simple form, it can be easily employed for the closed-loop control strategies such as the linearizing feedforward-feedback control method\(^{33}\) and RBF network inverse control schemes.\(^{34}\)

A linear approximation of the nonlinear system is computed using an RLS algorithm to form the LDB, which is characterized using the Laguerre filters. In other words, the proposed method is considered as a block-oriented identification method, starting from a linear approximation of the nonlinear system. A good survey on this subject can be found in the work of Schoukens and Tiels.\(^{35}\)
The difference between the system outputs in (6) and the Wiener model outputs in (7) is considered as the modeling error \( e(k) = y(k) - \hat{y}(k) \). The goal is to minimize the mean square of this error in the identification procedure.

The proposed method is composed of two phases: offline and online. In the offline phase, for each subsystem, the Laguerre filter parameter \((a_i)\) is computed using (5). It should be noted that the time constant of the system can be determined from the step response with small amplitude. If the step response of the system is not available or cannot be obtained, a linear first-order approximation of the system can be made using the measured input/output signals. This linear model can be determined in one of the known data-based linear identification procedures. For example, subspace identification methods are good choices for this matter. Then, computing the time constant of the system from the first-order approximation is straightforward. Using (4), the state matrices \((A_i)\) and the input matrices \((B_i)\) of the LDB can be calculated.

In the online phase, the output vector of the LDB \((C_i)\) and the synaptic weights of the NSB \((w_i)\) are adjusted for each subsystem via the measured input-output signals acquired from the system. To this end, an RLS algorithm is developed to update \( C_i \) in the LDB while a Lyapunov-based approach is utilized as the adaptation law for \( w_i \) in the MLP NN. More precisely, after determination of the LDB structure in the offline phase, the output vectors are tuned as follows:

\[
C_i(k) = C_i(k-1) + g_i(k) \left[ y_i(k) - C_i(k-1)I_i(k) \right], \quad i = 1, \ldots, m
\]

where

\[
g_i(k) = \left( 1 + I_i^T(k)P_i(k-1)I_i(k) \right)^{-1}P_i(k-1)I_i(k)
\]

in which

\[
P_i(k) = \frac{1}{\mu_i} \left[ 1 - I_i^T(k)g_i(k) \right] P_i(k-1),
\]

where \( \mu_i \) denotes the forgetting factor and \( P_i(0) \) is the square identity matrix. The adaptation law for the weights of the MLP NN is defined as follows:

\[
w_i(k+1) = w_i(k) + \zeta_i(k+1)\epsilon_i(k+1), \quad i = 1, \ldots, m,
\]

where

\[
\epsilon_i(k+1) = y_i(k+1) - \hat{w}_i^T(k)\Phi_i(k+1)
\]

\[
\zeta_i(k+1) = \frac{\Phi_i(k+1)}{\|\Phi_i(k+1)\|^2} \left( 1 - \eta_i \frac{|\epsilon_i(k)|}{\gamma + |\epsilon_i(k+1)|} \right),
\]

in which \( \epsilon_i(k) = y_i(k) - \hat{y}_i(k) \) is the modeling error and \( \eta_i \) is the learning rate. The small positive constant \( \gamma \) is only added to avoid division by zero. The block diagram of the proposed identification procedure is shown in Figure 2.
4 | ANALYTICAL STUDIES

As it was mentioned in Section 2, a linear system with input time delay can be modeled with a bank of Laguerre filters. In the following, Theorem 1 shows the upper-bound error for this approximation. Moreover, Theorem 2 shows convergence of the Wiener model outputs to the nonlinear system outputs.

**Theorem 1.** A strictly proper discrete-time linear time-invariant (LTI) system described by $G(z)$ that is analytic in $|z| > 1$ and continuous in $|z| \geq 1$ can be approximated by a sequence of Laguerre filters with the following error bound:

$$\left| G(z) - \sum_{k=1}^{N} c^{(k)} L^{(k)}(z) \right| \leq \sqrt{\frac{(1 + a)T_s}{1 - a}} \sum_{k=N+1}^{\infty} |a^{(k)}|. \quad (13)$$

**Proof.** Using (2), the Laguerre transfer functions can be rewritten in a recursive form as

$$L^{(1)}(z) = \sqrt{\frac{(1 - a^2)T_s}{(z - a)}}$$

$$L^{(i)}(z) = L^{(1)}(z) \left( \frac{1 - az}{z - a} \right)^{i-1}, \quad i = 1, \ldots, N. \quad (14)$$

As a result, the magnitude of the frequency response of the Laguerre transfer functions is

$$\left| L^{(i)}(e^{j\omega}) \right| = \sqrt{\frac{(1 - a^2)T_s}{e^{j\omega} - a}} \cdot \left( \frac{1 - ae^{j\omega}}{e^{j\omega} - a} \right)^{i-1}. \quad (15)$$

Based on the fact that $|\frac{1-ae^{j\omega}}{e^{j\omega}-a}| = 1$, it is can be inferred

$$\sup_{\omega} \left| L^{(i)}(e^{j\omega}) \right| = \sqrt{\frac{(1 + a)T_s}{1 - a}}. \quad (16)$$
On the other hand, every discrete-time function that is analytic in $|z| > 1$ and continuous in $|z| \geq 1$ can be represented exactly as

$$G(z) = \sum_{i=1}^{\infty} a^{(i)}L^{(i)}(z),$$  \hspace{1cm} (17)

where the Laguerre filter parameter is assumed to be $a < 1$. With the expansion at hand, the identification error can be stated as

$$\left| G(z) - \sum_{i=1}^{N} c^{(i)}L^{(i)}(z) \right| \leq \sum_{i=1}^{N} |a^{(i)} - c^{(i)}|L^{(i)}(z) + \sum_{i=N+1}^{\infty} a^{(i)}L^{(i)}(z).$$  \hspace{1cm} (18)

For brevity, the first term on the right-hand side of (18) is denoted as Term1 and the second one as Term2.

Next, consider the following Lyapunov function:

$$V(k) = \mathbf{c}^T(k)P_1(k)^{-1}\mathbf{c}(k),$$  \hspace{1cm} (19)

where $\mathbf{c}(k)$ is the error vector for the Laguerre filter coefficients of Term1 described as

$$\mathbf{c}(k) = \begin{bmatrix} a^{(1)}(k) - c^{(1)}(k) & \cdots & a^{(N)}(k) - c^{(N)}(k) \end{bmatrix}^T.$$  \hspace{1cm} (20)

Using (8)-(10) and (20), the first difference of (19) can be written as

$$V(k) - V(k-1) = \frac{1}{1 + \mathbf{1}^T(k)\mathbf{P}_1(k-1)\mathbf{1}(k)}\left[ y_1(k) - C_1(k-1)\mathbf{1}(k) \right]^2.$$  \hspace{1cm} (21)

Substituting (10) and (19) in (21), the Lyapunov function becomes

$$V(k) = \mu_i(k)V(k-1) - \mu_i(k)\left[ y_1(k) - C_1(k-1)\mathbf{1}(k) \right]^2.$$  \hspace{1cm} (22)

Therefore,

$$V(k) \leq \mu_i(k)V(k-1).$$  \hspace{1cm} (23)

Hence, the rate of convergence of $V(k)$ can be controlled with $\mu_i(k)$. A relatively small forgetting factor $\mu_i(k)$ forces Term1 to converge to zero.

Next, according to (16), for Term2, it can be written as

$$\sum_{i=N+1}^{\infty} a^{(i)}L^{(i)}(z) \leq |L^{(i)}(z)|\sum_{i=N+1}^{\infty} |a^{(i)}| \leq \sqrt{\frac{(1 + a)T_d}{1 - a}} \sum_{i=N+1}^{\infty} |a^{(i)}|.$$  \hspace{1cm} (24)

For a linear function that is continuously Lipschitz of order more than 0.5, it has been shown that

$$\sum_{i=1}^{\infty} |a^{(i)}| \leq \infty.$$  \hspace{1cm} (25)

Hence, the linear systems can be identified with an upper bounded error indicated in (13).

\textbf{Remark 1.} The sole objective of Theorem 1 is to demonstrate that a lattice of Laguerre filters can effectively approximate the response of a linear system with input time delay. This capability is demonstrated in the first simulation example in Section 4. Theorem 1 shows the upper-bound error for this approximation.

\textbf{Remark 2.} As it was explained in Section 2.1, the Laguerre filter parameter $(a)$ can be determined using (5). The time constant of the system $(T)$ can be computed as

$$T = T_1 + T_d,$$  \hspace{1cm} (26)

where $T_1$ is the time constant without the time delay and $T_d$ is the time delay in the step response. As (26) shows, increment in the input time delay $(T_d)$ increases the total time constant $(T)$, which in turn increases the Laguerre filter parameter $(a)$ according to (5). Then, based on (13), the upper bound of the identification error increases. In other words, larger input time delays can yield larger modeling error with the same sampling time.

\textbf{Remark 3.} Based on Theorem 1, a very large time delay not only reduces the stability margin but also increases the modeling error. However, due to the exponential form of (5), this increment does not grow linearly with $T_d$. On the other hand, the positivity of the Laguerre filter parameter $(a)$ avoids oscillatory response of the model.
Remark 4. As (13) indicates, the error bound of the linear system with input time delay is directly related to the sampling time \( T_s \). In other words, smaller sampling times yield smaller modeling error and vice versa. In addition, based on the Nyquist (Shannon) theorem, in order to reconstruct the continuous signal from the sampled data, the sampling frequency should satisfy the following condition:

\[
fs > 2f_{\text{max}},
\]

where \( f_{\text{max}} \) is the maximum frequency in the required bandwidth of the main system. However, due to practical considerations, higher sampling frequency should be selected.

**Theorem 2.** Consider the nonlinear system described in (6) and the Wiener model defined in (7). Convergence of the Wiener model outputs \( \hat{y}(k) \) to the system outputs \( y(k) \) is guaranteed if the weights of the MLP NN are updated using (11).

**Proof.** Consider the following discrete-type Lyapunov function:

\[
E(k) = \frac{1}{2} e^T(k) e(k) = \frac{1}{2} (y(k) - \hat{y}(k))^T (y(k) - \hat{y}(k)) > 0, \tag{27}
\]

where \( e(k) = [e_1(k) \cdots e_m(k)] \), \( y(k) = [y_1(k) \cdots y_m(k)] \), and \( \hat{y}(k) = [\hat{y}_1(k) \cdots \hat{y}_m(k)] \) are the output error vector and the system and the Wiener model outputs, respectively. The first difference of (27) is

\[
\Delta E(k) = E(k+1) - E(k) = \frac{1}{2} [e^T(k+1)e(k+1) - e^T(k)e(k)] \tag{28}
\]

that can be expressed as

\[
\Delta E(k) = \frac{1}{2} \sum_{i=1}^{m} [e_i^2(k+1) - e_i^2(k)] . \tag{29}
\]

Substituting \( e_i(k) = y_i(k) - \hat{y}_i(k) \) in (29) and using (7) and (11) yield

\[
\Delta E(k) = \frac{1}{2} \sum_{i=1}^{m} \left\{ [y_i(k+1) - w_i^T(k+1)\Phi_i(k+1)]^2 - e_i^2(k) \right\}
\]

\[
= \frac{1}{2} \sum_{i=1}^{m} \left\{ [y_i(k+1) - (w_i(k) + \zeta_i(k+1)e_i(k+1))^T\Phi_i(k+1)]^2 - e_i^2(k) \right\} . \tag{30}
\]

Using (12), (30) can be written as follows:

\[
\Delta E(k) = \frac{1}{2} \sum_{i=1}^{m} \left\{ [y_i(k+1) - w_i^T(k)\Phi_i(k+1) + e_i(k+1)\zeta_i^T(k+1)\Phi_i(k+1)]^2 - e_i^2(k) \right\}
\]

\[
= \frac{1}{2} \sum_{i=1}^{m} \left\{ \epsilon_i(k+1) - \epsilon_i(k+1) \left[ \Phi_i(k+1) \frac{1}{\|\Phi_i(k+1)\|^2} \left( 1 - \eta_i\frac{|e_i(k)|}{|e_i(k+1)|} \right) \right]^T \Phi_i(k+1) \right\}^2 - e_i^2(k) \tag{31}
\]

since \( \Phi_i^T(k+1)\Phi_i(k+1) / \|\Phi_i(k+1)\|^2 = 1 \), (31) can be rewritten as

\[
\Delta E(k) = \frac{1}{2} \sum_{i=1}^{m} \left\{ \epsilon_i(k+1) - \epsilon_i(k+1) \left( 1 - \eta_i\frac{|e_i(k)|}{|e_i(k+1)|} \right) \right\}^2 - e_i^2(k) \tag{32}
\]

which can be simplified to

\[
\Delta E(k) = \frac{1}{2} \sum_{i=1}^{m} \{ (\eta_i - 1)e_i^2(k) \} . \tag{33}
\]

Therefore,

\[
\Delta E(k) = -\frac{1}{2} e^T(k)(I - \eta)e(k) , \tag{34}
\]

where \( \eta = \text{diag}\{\eta_i^2\} \in \mathbb{R}^{m \times m} \) and \( I \in \mathbb{R}^{m \times m} \) is the identity matrix. Hence, \( \Delta E(k) \) is negative definite when all learning rates are bounded as \( 0 < \eta_i < 1 \). \qed
Remark 5. As it was proved in Theorem 2, convergence of the model output to the system output can be guaranteed when the learning rate is selected in the range of $0 < \eta_i < 1$. Based on the system structure, it is common to fine-tune the parameters of the NN using experience and some trial-and-error methods. Nevertheless, the proposed method is not very sensitive to the value of $\eta_i$. Only one-decimal-point precision suffices.

Discussion. As it is stated in Section 3, a linear approximation of the main nonlinear system is computed using the RLS to form the LDB. Theorem 1 only shows that the LDB, which is characterized using the Laguerre filters, can model the linear approximation of the system with bounded modeling error. The sole objective of Theorem 1 is to demonstrate that a lattice of Laguerre filters can effectively describe a linear system with input time delay. On the other hand, in the case of the Laguerre-MLP Wiener structure, Theorem 2 shows that the output of the model always converges to the system output because the LDB is BIBO stable, the output of the sigmoidal function is bounded between $-1$ and $1$, and the synaptic weights of the NSB is trained adaptively using the Lyapunov-based approach.

5 | CASE STUDIES

In this section, performance of the proposed identification method is demonstrated using four simulation examples. The first example is considered to show the capability of the Laguerre filters to model an LTI system with input time delay. The next two examples describe practical nonlinear process systems. These examples show that the proposed identification method can model SISO and MIMO systems with input time delay with very good accuracies in the open-loop mode. The last case study is considered to show the property of the proposed algorithm for Example 2 in the closed-loop control application.

It should be noted that the input time delays are known a priori just to show the accuracy of the proposed method. There is no need to know the value of the time delays in the proposed algorithm.

5.1 | Linear system with input time delay

This case study is intended only to show validity of Theorem 1 in modeling linear systems with input time delay using Laguerre filters.

The signal attenuation in a certain communication channel is described by a linear system introduced by a continuous-time state-space representation as follows:

$$A = \begin{bmatrix} -0.8147 & 0 \\ 0.9058 & -0.1270 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0.1904 \end{bmatrix}, \quad D = 0. \quad (35)$$

This system has an input time delay that is equal to $T_d = 8$ seconds. This system is simulated for 200 seconds with the sampling time equal to 0.1 seconds. The Laguerre filter parameter is determined using the unit step response of the system in the offline stage. The system time constant is 20 seconds. (Figure 3). Hence, $a = 0.9950$.

Figure 3 shows the step response of the system and the Laguerre filter model for different values of $N$ in (3). As it was mentioned in Section 1, this value can be less than or equal to the system order in the state space representation. For this system, $N = 2$ yields the best response of the filter.

5.2 | Continuous-stirred tank reactor with input time delay

An exothermic reaction, which is carried out in a continuous-stirred tank reactor (CSTR) can be mathematically modeled in the following dimensionless form:

$$\begin{cases}
\dot{x}_1(t) = -x_1(t) + D_a (1 - x_1(t)) \exp\left(\frac{x_2(t)}{1 + (x_2(t)/\phi)}\right) \\
\dot{x}_2(t) = -x_2(t) + BD_a (1 - x_1(t)) \exp\left(\frac{x_1(t)}{1 + (x_1(t)/\phi)}\right) - \beta_r (x_2(t) - x_3(t)) \\
\dot{x}_3(t) = \beta_r (x_2(t) - x_3(t)) + (x_{sf} - x_3(t)) u(t - \tau) \\
y(t) = x_2(t)
\end{cases} \quad (36)$$
FIGURE 3  Step response of communication channel system and Laguerre model for A, $N = 1$; B, $N = 2$; C, $N = 5$ [Colour figure can be viewed at wileyonlinelibrary.com]
where $x_1$, $x_2$, and $x_3$ denote the dimensionless reactant concentration, reactor, and cooling jacket temperatures, respectively, and $u$ is the flow rate of the input cooling liquid with time delay $\tau$. The other parameters are shown in Table 1 for the system’s working point. The desired output is the reactor temperature ($x_2$).

In this system, $T_1 = 1$ minute, $T_d = 5$ minutes, and $T_s = 0.1$ minutes. Therefore, $a = 0.9835$. The LDB is composed of a lattice with two Laguerre filters. The NSB is made up of an MLP NN that is composed of one hidden layer with three neurons. The activation function of neurons is of tangent hyperbolic type $\varphi(x) = (1 - e^{-0.5x})/(1 + e^{-0.5x})$. The synaptic weights are randomly selected at the beginning of the identification task. The learning rate $\eta$ is equal to 0.1.

An input-output data set is gathered from the computerized simulation of the CSTR system for 75 minutes. Figure 4 demonstrates the capability of the proposed method as compared with the Hammerstein fuzzy-Laguerre and the Wiener Laguerre-wavelet identification procedures represented in the works of Aadaleesan et al.\textsuperscript{27} and Zhao et al.\textsuperscript{43} respectively. The reason for comparison with the aforementioned works\textsuperscript{27,43} is as follows. The Laguerre-wavelet Wiener procedure in the work of Aadaleesan et al.\textsuperscript{27} has adopted a similar block-oriented structure as our proposed Laguerre-MLP Wiener form. Moreover, Aadaleesan et al.\textsuperscript{27} and Zhao et al.\textsuperscript{43} have employed Laguerre filters to describe the LDB in order to compensate for the delay time. On the other hand, the method presented in the work of Zhao et al.\textsuperscript{43} which employs the fuzzy logic and Laguerre filters in the Hammerstein model, is one of the advance works in the area of identification of nonlinear systems with input time delay using the block-oriented structure.

The root-mean-squared error (RMSE) of the proposed modeling method is equal to 0.0024 while this value for the fuzzy-Laguerre method and the Laguerre-wavelet approach is equal to 0.3464 and 0.1500, respectively. Moreover, the fuzzy-Laguerre and the Laguerre-wavelet models have a large number of adjustable parameters and their modeling performances are too sensitive to the selected structures for NN, system nonlinearity, and the initial values. The output of the Laguerre filters network is shown in Figure 4, which clearly indicates the importance of the MLP NN on the modeling accuracy.

Figure 5 shows the output of the proposed Wiener model using $\eta = 1.05$, which is greater than the bound necessary for convergence. As this figure shows, a negligible increase in the learning rate higher than one yields undesirable effects on

### Table 1
Parameters of continuous-stirred tank reactor plant

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi$</td>
<td>20</td>
</tr>
<tr>
<td>$B$</td>
<td>11</td>
</tr>
<tr>
<td>$D_a$</td>
<td>0.14</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.5</td>
</tr>
<tr>
<td>$x_{3f}$</td>
<td>$-4$</td>
</tr>
</tbody>
</table>
the convergence of the model output. Higher values of $\eta$ would result in unstable outputs. These results are in agreement with Theorem 2.

Table 2 shows a quantitative comparison between the proposed method and the method presented in the works of Aadaleesan et al.\textsuperscript{27} and Wu and Chou.\textsuperscript{42}

### 5.3 Three-tank system

This system consists of three cylindrical tanks all with cross section $A$. These tanks are linked together through connective pipes with the cross section $S_n$. The out-flowing liquid of Tank II is collected in a storage tank, which feeds Pumps 1 and 2.\textsuperscript{44} The schematic diagram of this system is shown in Figure 6.

The pump flow rates $Q_1$ and $Q_2$ are the input signals and the liquid levels in Tanks 1 and 2 denote the output signals, respectively. Hence, this is a two-input–two-output system. The system is modeled in the nonlinear state space form as
follows:
\[
\begin{align*}
\dot{x} &= f(x) + Bu(t - \tau_1) \\
y &= C^T x,
\end{align*}
\]
where the state space variables are determined as
\[
x := \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}, \quad u = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix},
\]
\[
f(x) = \frac{1}{A} \begin{bmatrix} -Q_{13} \\ Q_{32} - Q_{20} \\ Q_{13} - Q_{32} \end{bmatrix}, \quad B = \frac{1}{A} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},
\]
in which the flow rates of the connective pipes are as follows:
\[
\begin{align*}
Q_{13} &= a_{z1} S_n g sgn(h_1 - h_3)(2g|h_1 - h_3|)^{1/2} \\
Q_{32} &= a_{z2} S_n g sgn(h_3 - h_2)(2g|h_3 - h_2|)^{1/2} \\
Q_{20} &= a_{z2} S_n (2gh_2)^{1/2},
\end{align*}
\]
where \( g \) determines the gravity acceleration, \( a_{z_i} \) and \( h_i \) are the outflow coefficients and liquid level of the \( i \)th tank, respectively, and \( sgn(\cdot) \) indicates the sign function. The parameters of the three-tank system for the nominal working point are given in Table 3.\(^4\)

In this system, the time constant without the time delay of the first and second input signals is equal to 10 and 15 seconds, respectively. The time delay for the first input line is 30 seconds and there is no time delay in the second input line. The sampling time is equal to 0.1 seconds. Hence, \( a_1 = 0.9975 \) and \( a_2 = 0.9934 \). In the Wiener structure, the LDB is composed of two separate lattices each with three Laguerre filters. The NSB is composed of two MLP NNs each with two neurons in the hidden layer with the same activation function as the CSTR system. The weights of the NNs are adapted using the proposed Lyapunov-based algorithm with the learning rates equal to 0.1 and 0.25, respectively. Figure 7 shows the input signals of the three-tank system during the online identification procedure. The output signals of the proposed Wiener model are shown in Figure 8 and are compared with the outputs of the Hammerstein fuzzy-Laguerre\(^4\) and the Wiener

<table>
<thead>
<tr>
<th>TABLE 3 Technical parameters of three-tank model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = 154 \text{ cm}^2 )</td>
</tr>
<tr>
<td>( H_{\text{max}} = 62 \text{ cm} )</td>
</tr>
<tr>
<td>( a_{z1} = 0.5 )</td>
</tr>
</tbody>
</table>

\( a_1 = 0.9975 \) and \( a_2 = 0.9934 \). In the Wiener structure, the LDB is composed of two separate lattices each with three Laguerre filters. The NSB is composed of two MLP NNs each with two neurons in the hidden layer with the same activation function as the CSTR system. The weights of the NNs are adapted using the proposed Lyapunov-based algorithm with the learning rates equal to 0.1 and 0.25, respectively. Figure 7 shows the input signals of the three-tank system during the online identification procedure. The output signals of the proposed Wiener model are shown in Figure 8 and are compared with the outputs of the Hammerstein fuzzy-Laguerre\(^4\) and the Wiener

![Figure 7 Input signals of the three-tank system](image-url)
Laguerre-wavelet\textsuperscript{27} models. It should be noted that the identification procedure represented in the work of Zhao et al.\textsuperscript{43} has been defined for MISO systems and it has been extended for the MIMO systems.

One of the main advantages of the proposed online identification algorithm is its robustness against changes in the system parameters. In this case study, the cross-section of the connective pipes ($S_n$) is altered at time $t = 480$ seconds from 0.5 to 0.05. As Figure 8 shows, the proposed adaptive method can follow the system outputs without any noticeable error, whereas the tracking capability of the other two offline methods has been declined drastically.

The acquired RMSE for the first and second output is equal to 0.1282 and 0.0668, respectively. The RMSE of the fuzzy-Laguerre and Laguerre-wavelet methods for the first output is equal to 35.7872 and 27.7738, respectively. Moreover, the RMSE of the second output for the fuzzy-Laguerre approach is 24.8844, whereas for the Laguerre-wavelet is calculated as 14.0279.

### 5.4 CSTR with input time delay in closed-loop control

The parameterized state-feedback control strategy is used for the closed-loop controller for the CSTR input time delay,\textsuperscript{42} which is described in (36). The proposed method linearizes the input-output behavior of the system such that the output-tracking problem is obtained (Figure 9).

![FIGURE 9 Closed-loop identification block diagram](image-url)

\textbf{FIGURE 8} Output signals of the three-tank system and the Wiener model [Colour figure can be viewed at wileyonlinelibrary.com]
The desired reference model can be depicted as follows:

\[
\begin{align*}
\dot{z}_1(t) &= z_2(t) \\
\dot{z}_2(t) &= -\zeta_1 z_1(t) - \zeta_2 z_2(t) + \zeta_1 y_{sp}(t) \\
y_d(t) &= z_1(t),
\end{align*}
\] (40)

where \(\zeta_1\) and \(\zeta_2\) are positive constants. In simulations, the following assumptions are considered:

\[y_{sp} = 4 \text{ and } (\zeta_1, \zeta_2) = (4, 4).\]

In this process, the main objective is to follow the desired trajectory that is defined for the reactor temperature.

**FIGURE 10** Input signal of continuous-stirred tank reactor in closed-loop control

**FIGURE 11** Output signal of continuous-stirred tank reactor in closed-loop identification [Colour figure can be viewed at wileyonlinelibrary.com]
According to the suggested control strategy in the work of Wu and Chou, the parameterized state feedback can be obtained using the following manipulated input:

\[
u(t) = \left( \beta_x (x_{3f} - x_3(t)) \right)^{-1} \begin{bmatrix}
\left[ BD_a w(x_2(t)) \right] \times [-x_1(t) + D_a (1 - x_1(t)) w(x_2(t))] \\
- \left[ -1 + BD_a (1 - x_1(t)) w(x_2(t)) (1 + x_2(t)/\phi^2) - \beta_x \right] \\
\times [-x_2(t) + BD_a (1 - x_1(t)) w(x_2(t)) - \beta_x (x_2(t) - x_3(t))] \\
- \beta_x [\beta_x (x_2(t) - x_3(t))] + \left[ -\zeta_1 z_1(t) - \zeta_2 z_2(t) + \xi_1 y(t) \right] \\
- \alpha_1 (x_2(t) - z_1(t)) \\
- \alpha_2 \left[ x_2(t) + BD_a (1 - x_1(t)) w(x_2(t)) - \beta_x (x_2(t) - x_3(t)) \right] - z_2(t) \end{bmatrix},
\]

where \( w(x_2(t)) = \exp (x_2(t)/(1 + x_2(t)/\phi)) \) and \( \beta_x (x_{3f} - x_3(t)) \) is nonzero to guarantee the feasibility of the feedback linearization. Throughout simulations, the fixed pole placement gains are selected as \( (\alpha_1, \alpha_2) = (0.01, 0.2) \). The manipulated input signal is shown in Figure 10.

In Figure 11, the output of the proposed identification approach is compared to the system output in the closed-loop identification scenario, which indicates good applicability of the proposed method in open-loop identification cases and in closed-loop controlled systems. Table 4 shows a qualitative comparison between the proposed method and the methods presented in the works of Aadaleesan et al and Wu and Chou.

### 6 CONCLUSIONS

This paper proposed a real-time identification method for modeling nonlinear MIMO systems with input time delay. The proposed approach employed the Laguerre filters as the linear-dynamic block and the MLP NN as the nonlinear-static block in the Wiener model. Using the Lyapunov direct method it was shown that the cascade of Laguerre filters could approximate linear systems with input time delay. Moreover, it was shown analytically that the outputs of the Wiener model converge to the system outputs. Online adaptation of the Wiener model parameters helped identification procedure to cope with changes in the system parameters. The capability of the suggested method verified in the closed-loop control task as well. Four practical simulation examples showed good modeling property of the proposed method as compared with the recently reported block-oriented algorithms in literature.

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